Notes 7.3: Double Angle Identities:

$$\sin 2\theta = 2\sin\theta\cos\theta$$

← Already on yellow note sheet

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 1 - 2\sin^2\theta$$
$$= 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$ *NEW*

Add to yellow sheet:

Half Angle Identities:

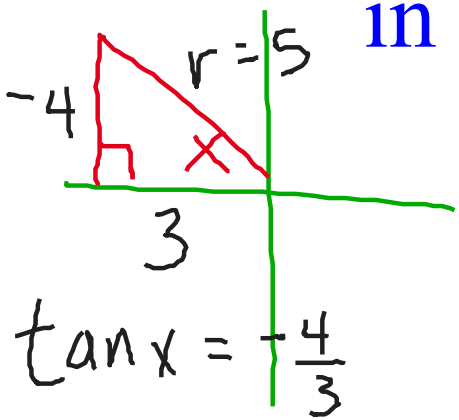
$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} \quad \text{or} \quad \frac{\sin x}{1+\cos x}$$

Important: Determine trig values for θ using given information. Let the formula determine final value of 2θ.

7.3 #4 Find sin2x, cos2x, tan2x given $\tan x = -\frac{4}{3}$ in Quad II.



$\tan x = -\frac{4}{3}$

$\sin x = \frac{-4}{5}$

$\cos x = \frac{3}{5}$

$\sin 2x = 2\sin x \cos x$

$= 2\left(\frac{-4}{5}\right)\left(\frac{3}{5}\right)$

$= \boxed{\frac{-24}{25}}$

$\cos 2x = \cos^2 x - \sin^2 x$

$= \left(\frac{3}{5}\right)^2 - \left(\frac{-4}{5}\right)^2$

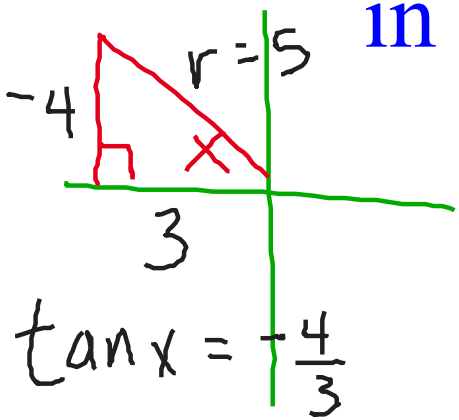
$= \frac{9}{25} - \frac{16}{25}$

$= \boxed{\frac{-7}{25}}$

tan 2x → next slide

Important: Determine trig values for θ using given information. Let the formula determine final value of 2θ.

7.3 #4 Find sin2x, cos2x, tan2x given $\tan x = -\frac{4}{3}$ in Quad II.



$\tan x = -\frac{4}{3}$

$\sin x = \frac{-4}{5}$

$\cos x = \frac{3}{5}$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\left(\frac{-4}{3}\right)}{1-\left(\frac{-4}{3}\right)^2}$$

$$= \frac{-\frac{8}{3}}{1-\frac{16}{9}}$$

$$= \frac{-\frac{8}{3}}{-\frac{7}{9}}$$

$$= -\frac{8}{3}\cdot\frac{9}{-7}$$

$$= \frac{24}{7}$$

Use a half angle identity to find the exact value:

7.3 #18 tan15°

$$\tan\left(\frac{30^\circ}{2}\right) = \frac{1-\cos 30^\circ}{\sin 30} = \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= \frac{2\left(1-\frac{\sqrt{3}}{2}\right)}{2\left(\frac{1}{2}\right)} = \frac{2-\sqrt{3}}{1} = \boxed{2-\sqrt{3}}$$

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} \quad \text{or} \quad \frac{\sin x}{1+\cos x}$$

$\theta=$	30°	45°	60°
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Use a half angle identity to find the exact value:

7.3 #22 cos112.5°

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}} \longrightarrow$$

Choose negative since 112.5° = Quad II

$$\cos\left(\frac{225°}{2}\right) = -\sqrt{\frac{1+\cos 225°}{2}}$$

Quad III ref angle is 45°

$$= -\sqrt{\frac{1+\frac{-\sqrt{2}}{2}}{2}}$$

$$= -\sqrt{\frac{\left(1-\frac{\sqrt{2}}{2}\right)2}{(2)2}}$$

$$= -\sqrt{\frac{2-\sqrt{2}}{4}}$$

square root separately →

or $2^{-\frac{1}{2}}$ $2-\sqrt{2}$

$$= \frac{-\sqrt{2-\sqrt{2}}}{2}$$

Important: Evaluate +/- for both the outside and inside (treat them as two separate values)